PART A

ANSWER ALL QUESTIONS (10 x 2 =20)

1. Is it possible to solve f(x) = $(x^2 - 2.4)^2$ using numerical methods.
2. Solve $\frac{dy}{dx} = y - \frac{2x}{y}$;y(0) =1 in the range 0≤ x ≤0.2 using Euler's method.
3. Expand f(z) = sin z in Taylor series about z = $\frac{\pi}{4}$.
4. Show that in general, $\int_c f(z)dz$ is dependent on the path followed.
5. Define the scalar product and inner product in dual vector space.
6. Show that the linear combination of symmetric matrices is a symmetric matrix.
7. Show that $\delta^i_j A^j = A^i$
8. If x^i and $\bar{x}^i$ are independent coordinates of a point, prove that

$$\frac{\partial x^i}{\partial \bar{x}^p}\frac{\partial \bar{x}^p}{\partial x^j} = \delta^i_j$$

9. Using the knowledge of beta function evaluate $\beta\left(\frac{1}{3}, \frac{2}{3}\right)$
10. Using Rodrigue's formula, obtain Legendre polynomial $P_2(x)$

PART B

ANSWER ANY FOUR QUESTIONS (4x7.5 = 30)

11. Solve $x^3 - 2x - 5 = 0$ using Newton- Raphson method
12. a) State and prove Cauchy-Riemann conditions from first principles.

    b) Prove Cauchy's theorem for analytic function.
13. Define a proper and improper rotation. Verify that the matrix A given below is a rotation matrix and obtain the corresponding axis and angle of rotation

$$A=\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

14. If 'p' be a point in the solid at which stress is to be specified, establish the Cartesian components of stress tensor.
15. Derive any two recurrence relations of Legendre polynomials.
16. Evaluate $\frac{1}{2\pi i}\oint \frac{\cos \pi z}{z^2-1}\, dz$ around a rectangle with vertices at (i) $2 \pm i$, $-2 \pm i$
    (ii) $-i, 2-i, 2+i, i$

PART C

ANSWER ANY FOUR QUESTIONS **(4 x 12.5 = 50)**

17. Using Gauss- elimination method, solve

$3.15x - 1.96y + 3.85z = 12.95$

$2.13x + 5.12y - 2.89z = -8.61$

$5.92x + 3.05y + 2.15z = 6.88$

18. Evaluate a) $\oint_c \frac{z\,e^z}{(4z+\pi i)^2}dz$ where c: $|z| = 10$

b) $\oint_c \frac{\cos^3 z}{\left(z+\frac{\pi}{6}\right)^2}dz$ where c: $|z| = 9$

19. a) Find an orthonormal set of basis vectors for the plane x + y + z = 1.

b) Using a certain basis in a three dimensional vector space, a linear operator is represented by a matrix A and a particular vector by the column vector V where $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$; $V = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

Find the matrix and column vector for the operator as well as the vector in a new basis represented by $u^{(1)} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$;$u^{(2)} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ and $u^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

20. a) Define metric tensor.

b) Obtain the metric tensor for a three dimensional Euclidean space in terms of spherical polar coordinates.

c) Obtain the contravariant components of the metric tensor in terms of spherical polar coordinates.

21. Solve Bessel's differential equation using Frobenius power series method.

22. i) Prove that $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$ m, n > 0

ii)Evaluate $\int_0^1 x^2(1-x)^3\,dx$ and $\int_0^2 (4-x^2)^{\frac{3}{2}}\,dx$